Loop Quantum Cosmology

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by TeX and GNU Emacs

Outline

- Quantum Gravity's importance;
- Present Approaches to Quantum Gravity;
- Basics of Loop Quantum Gravity;
- LQG Results;
- Loop Quantum Cosmology; &
- LQC Recent Applications.

Quantum Gravity: Why is it important?! ☻

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Relation to other problems

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Relation to other problems

- Quantum Cosmology & Origin of the Universe;
- Quantum Theory "without time", Unitarity;
- Structure and Interpretation of Quantum Mechanics, Topos theory;
- Wave Function Collapse;
- Unification of All Interactions, TOEs;
- Final State of a Black Hole; &
- Ultraviolet Divergences.

Present Approaches to Quantum Gravity

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Main Directions

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Main Directions

- **String Theory**; &
- **Loop Quantum Gravity**.

Traditional Approaches

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- **Discrete Approaches** (Regge calculus, dynamical triangulations);
- **Old Hopes, Approximate Theories** (Euclidean QG, QFT on curved spacetime); &
- **"Unorthodox Approaches"** (causal sets, twistors, Finkelstein's ideas).

New Directions

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- **Noncommutative Geometry**;
- **Null Surface Formulations**; &
- **Spin Foam Models** (TQFTs).

Loop Quantum Gravity: The Basics

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- **Quantum Field Theory on a Differentiable Manifold**

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  ⋆ background independence;

  ⋆ pseudo-1-form (densitized triad); &

  ⋆ $SU(2)$-connection.

Loop Quantum Gravity: The Basics (cont'd)

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- **Wilson Loop Operators**
  - ⋆ **well defined** in the Hilbert space of the theory; &
  - ⋆ non-perturbative.
- **Physical meaning of diffeomorphism invariance and its implementation in the quantum theory**
  - ⋆ **all** variables are **dynamical**!

LQG: Results

- **Technical**

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  - ⋆ Solution of the Hamiltonian constraint
  - ⋆ Time Evolution, Topological Feynman rules
  - ⋆ Fermions
  - ⋆ Maxwell & Yang-Mills
  - ⋆ Lattice & Simplicial models

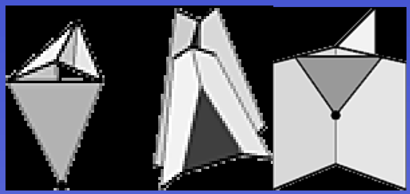
LQG: Results (cont'd)

- **Physical**

LQG: Results (cont'd)

- **Physical**

  ⋆ Planck Scale Discreteness of space (area or volume operators have a discrete spectrum)



  ⋆ Classical Limit, quantum states for flat spacetimes

  ⋆ Black Hole Entropy

Loop Quantum Cosmology

- Hamiltonian constraint:

Loop Quantum Cosmology

- Hamiltonian constraint:

$$\frac{12\,\sqrt{|p|}}{\gamma^2}\left[c\,(c-k)+(1+\gamma^2)\,\frac{k^2}{4}\right]=8\,\pi\,G\,H_m(p,\phi,p_\phi)$$

where $|p| = a^2$, $c = 1/2\,(k - \gamma\,\dot{a})$, $k = 0, 1$ for a flat or closed model, H_m is the matter Hamiltonian. (p and c are part of the **isotropic** triad and connection, respectively.)

LQC (cont'd)

- This is just Friedman's eq,

$$3\,(\dot{a}^2 + k^2)\,a = 8\,\pi\,G\,a^3\rho_m(a,\phi,p_\phi)$$

- LQC: **symmetric** LQG states.

LQC (cont'd)

- Discrete Evolution

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LQC (cont'd)

- Discrete Evolution
  - ⋆ **Indirect** quantization: via holonomies, i.e., $\nexists\, c$ operator, just a holonomy related to it.
  - ⋆ Quantized Friedman eq: **difference** eq!
  - ⋆ $\Rightarrow$ **no** cosmological singularities (homogeneous models)!

LQC (cont'd)

- Finite Inverse Scale Factor Operator

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  - ⋆ Matter Hamiltonian: need to quantize a^{-3}
    - ∗ Inverse power of $\hat{p}$: discrete spectrum with 0
    - ∗ Alternative way yields 3 phases:

LQC (cont'd)

- Finite Inverse Scale Factor Operator
  - ⋆ Matter Hamiltonian: need to quantize a^{-3}
    - ∗ Inverse power of $\hat{p}$: discrete spectrum with 0
    - ∗ Alternative way yields 3 phases:
      1. $\hat{H}_m(0) = 0$
      2. Inflation
      3. Transition to the classical a^{-3}

Recent Applications

- Quantum Structure of Classical Singularities

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  ⋆ All homogeneous models: no cosmological singularities!

  ⋆ Inhomogeneous models: can be treated in the BKL scenario.

Recent Applications (cont'd)

- Phenomenology

Recent Applications (cont'd)

- Phenomenology
  - ⋆ Inflation:
    - ∗ **No** graceful exit problem (phase transition on the scale factor ⇒ classical regime);
    - ∗ Exists for **any** matter content, even without an inflaton (spacetime property).

Recent Applications (cont'd)

- Phenomenology
  - ⋆ Inflation:
    - ∗ **No** graceful exit problem (phase transition on the scale factor ⇒ classical regime);
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  - ⋆ Quantized spacetime: photons with different energies should have slightly different speeds (modified dispersion relations; gamma & cosmic rays).

Recent Applications (cont'd)

- Perturbative Corrections

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  - ⋆ Leading order: Friedmann eq (as seen); &
  - ⋆ Two types of corrections:

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- Perturbative Corrections
  - ⋆ Leading order: Friedmann eq (as seen); &
  - ⋆ Two types of corrections:
    1. Higher curvature corrections ($\simeq$ effective action); &
    2. Other terms: reflect that $\nexists$ unique coherent state such as the Minkowski vacuum for an effective action.

Bibliography

- *"Throwing Einstein for a Loop"*;
- *"Atoms of Space and Time"*;
- *"A Spin on Spin Foam"*;
- *"The Future of String Theory – A Conversation with Brian Greene"*;
- *"How far are we from the quantum theory of gravity?"*;
- *"Strings, loops and others: a critical survey of the present approaches to quantum gravity"*;
- *"Loop Quantum Gravity"*;
- *"Loop quantum gravity and quanta of spacetime: a primer"*;
- *"Lectures on Loop Quantum Gravity"*;
- *"Canonical quantum gravity and consistent discretizations"*;
- *"Absence of Singularity in Loop Quantum Cosmology"*;
- *"Quantum Geometry in Action: Big-Bang and Black Holes"*;
- *"Cosmological applications of loop quantum gravity"*; &
- *"Loop Quantum Cosmology: Recent Progress"*.

Just in case...

Oh, you thought I wouldn't be prepared for some questions, huh?! ☺

Not at all: In what follows there are some slides to entertain the more curious characters...

Appendix 1: Loop Quantization

- Quantum variables:
  - ⋆ Holonomies: $h_e(A) = \mathcal{P}\exp\{\int_e A_a^i\, \dot{e}^a\, \tau_i\, dt\} \;\in SU(2)$
  - ⋆ Fluxes: $F_S(E) = \int_S E_i^a\, n_a\, \tau^i\, d^2y$

- τ_i: Pauli matrices; $\dot{e}^a$: tangent vector to the edge e; n_a: co-normal to the surface S.

- If all curves e and all surfaces Σ are allowed, holonomies and fluxes contain the same information as the original fields.

Appendix 1: Loop Quantization (cont'd)

- Holonomies and fluxes are better for quantization: smeared versions of the fields obtained by natural integration along curves and surfaces

- One- and two-dimensional smearings can be done without introducing a background metric! Background independent quantization! ☺

- Quantum theory: defined on a representation of the background independent holonomy-flux Poisson $\star$-algebra

Appendix 1: Loop Quantization (cont'd)

- The basis for the states is given by **spin network** states which are associated to graphs in Σ whose edges are labeled by irreducible $SU(2)$ representations

- Diffeomorphism-invariant inner product: Ashtekar--Lewandowski

Appendix 1: Loop Quantization (cont'd)

- Characteristics of the Loop Quantization:
  - ⋆ Hilbert space before imposing the [diffeomorphism invariance and Hamiltonian] constraints is non-separable: all spin network states in different graphs are orthogonal to each other!
  - ⋆ Holonomies are well defined operators by definition!
  - ⋆ Flux operators have discrete spectra ⇒ discrete spatial geometry!