Branes: An Introduction

Part I

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Outline:

- Overview;

- Review of Basic Results:
  - Closed String Quantization;
  - Open String;

- T-duality:
  - Closed String;
  - Open String;
  - Unoriented String.
Overview:

“Once upon a time…” the world of physics was divided into two categories:

- **Small**: Quantum Mechanics, Quantum Field Theory;

- **Large**: General Relativity (Einstein’s theory of gravity).

Nowadays, those two pillars of modern physics are in conflict… One that is the modern version of earlier contradictions that lead to radical changes in the 20th century physics!
Overview: (cont’d)

Back in the “dark days”, the inverse square law of gravity and electromagnetism where very much alike:

\[ F_g = G \frac{M m}{r^2}, \quad F_e = K \frac{Q q}{r^2}. \]

This worked just fine for about 200 years... 😊 [It’s worth noting at this point that both of those “laws” follow from Gauss’ Law and spherical symmetry! Only the charges and the proportionality constants are different.]

However, when the electron was discovered... hell broke loose! 😊 And, ever since, physicists have had to deal with the singularity at \( r = 0! \)
Overview: (cont’d)

This seemed to imply that, e.g., an atom would survive for about $10^{-9}$ seconds!

However, this problem was solved with the aid of the uncertainty principle!

What it does is that it “smears out” the singularity at the origin (everything gets a bit fuzzy).

It turns out that this just doesn’t do it for gravity! 😊 The analogy between gravity and electromagnetism isn’t as close as the inverse square law suggests... And, in actuality, since 1916, gravity is described by Einstein’s General Relativity. (Thus, there is no inverse square law!) And, basically, the non-linearity of GR is what prevents the uncertainty principle from working.
**Overview:** (cont’d)

This leads us to **string theory**: During the 70’s, physicists discovered that this problem can be overcome in the stringy framework.

Roughly speaking, one reinterprets an elementary particle as a vibrating string/loop. The “roughly” is because (among other things) one should include **quantum uncertainty** on both of them: the particle and the string side!

Strings: many modes of oscillation/harmonics (representing different particles) ⇒ single string describes all (a huge assortment of particles) at once!

∴ upon quantization of the string and computation of the “spectrum” ⇒ find that one of these particles has the properties to be a “**graviton**”, the quantum of gravity!
Overview: (cont’d)

- Conventional QFT: Gravity is **impossible**;

- String Theory:
  - Gravity is **required**!
  - Other modes: photon, electron, muon, quarks, neutrinos, etc...

That is: Very simple assumptions ⇒ “everything” neatly fits into a string! 😊
Overview: (cont’d)

Among the fundamental differences between [ordinary] quantum theory and string theory, these are eye-popping ones:

- **Vertices:**
  - QFT: the interactions have to be described,
  - String: once the motion of a free string is understood, understanding the interactions is automatic, since the picture looks everywhere locally the same.

- **Infinities:**
  - QFT: Coalescing of the vertices $\Rightarrow$ product of $\delta$-functions (iff inverse square law — point-like interactions).
  - String: **no vertices** $\Rightarrow$ finite!
Overview: (cont’d)

• **QFT:**
  - Free motion: doesn’t really contain much information about spacetime;
  - Interaction (vertex): depends on knowing exactly where (product of $\delta$-functions) they happen!

• **String:** this last piece is missing $\Rightarrow$ never learns precisely what the spacetime is: New source of fuzziness that goes beyond quantum mechanical uncertainty,
  - $\hbar$: quantum mechanical $\rightarrow$ bound on the precision of “trajectory of a particle”,
  - $\alpha$: size of string $\rightarrow$ bound on the description of spacetime!
Overview: (cont’d)

String theory: 3 general predictions:

1. Gravity (Einstein’s GR);

2. Gauge Symmetry (bread ’n butter of QFT);

3. SuperSymmetry:
   - alongside the usual (“bose”) dimensions of spacetime \((t, x, y, z)\), there would be “fermi” dimensions, \(\theta^\alpha\). It predicts new particles \(\leftrightarrow\) oscillations in the dimension.

One does need some knowledge about the vacuum, though.
Overview: (cont’d)

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<td><strong>Old</strong> QUANTUM MECHANICS</td>
<td><strong>GR and QFT</strong></td>
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<tr>
<td><strong>New</strong> STRING THEORY</td>
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This problem has been around for quite a while... but its outlook has changed drastically in the last couple of years: **Duality!!!**

Duality:

- **Old version:** Symmetry between \( \vec{E} \) and \( \vec{B} \) which holds in vacuum \((c = 1)\),

\[
\nabla \cdot \vec{E} = 0 , \quad \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} ,
\]

\[
\nabla \cdot \vec{B} = 0 , \quad \nabla \times \vec{B} = \frac{\partial \vec{E}}{\partial t} ,
\]

\( \vec{E} \leftrightarrow \vec{B} \), \( \vec{B} \leftrightarrow -\vec{E} \).
Overview: (cont’d)

It is spoiled in nature due to the absence of a magnetic monopole.

Quantum Mechanics seems to make duality impossible: Role of the vector potential, $\vec{A}$.

Olive & Montonen: strong/weak coupling duality! (Swaping the electric charge with a magnetic monopole — supposed to exist.)

Limitations in our abilities in physics: “We are able to compute what happens only when the charge, $e$, is much less than one!” — Perturbation Theory!

Duality: exchange $e$ and $1/e$ — both of them cannot be much less than one!

New understanding of duality (non-perturbative): Severe implications for QFT and String Theory.
Overview: (cont’d)

E.g.:

- **QFT**: Quark confinement (related to more standard physics via duality — strong vs weak coupling),

- **ST**: There is only one **ST***!!! 😊 ⇒ Type I, Type IIA, Type IIB, Heterotic SO(32), \(E_8 \times E_8\) and 11-dim SUGRA are different limiting cases of one theory.
Overview: (cont’d)

Better understanding of String Theory: *Strings are the “first among equals” and share the stage with “D-branes” — they are related by duality!*

D-branes successes:

1. Black Hole entropy;

2. SuperSymmetric field theories;

3. Non-supersymmetric field theories;


5. Non-commutative spacetime...
**Closed String Quantization:** (quick review)

- **Worldsheet action:**

\[
S = -\frac{1}{2} \frac{1}{2\pi \alpha'} \int_{\mathcal{W}} \sqrt{-g} \ g^{ij} \partial_i X^\mu \partial_j X^\nu \eta_{\mu\nu} \ dt \ d\sigma ;
\]

where \((1/2\pi \alpha')\) is the string tension; \(i,j = 0,1\) are the worldsheet coordinates \((\tau, \sigma)\); and \(X^\mu\) are the spacetime coordinates, with \(\mu, \nu = 0, \ldots, D - 1\). (This can also be understood as 2-dimentional gravity coupled to "matter" \(X^\mu\).)

- 2-dim reparametrization invariance: Just like in GR, \(\sqrt{-g} \ d\tau \ d\sigma\) is the invariant volume and \(g^{ij} \partial_i X^\mu \partial_j X^\mu\) is properly contracted.

- **Conformal (Weyl) invariance:** \(g_{ij} \mapsto f(\sigma) g_{ij} \Rightarrow \sqrt{-g} \ g^{ij} \mapsto f^{\frac{1}{2}(n+1)-1} \sqrt{-g} \ g^{ij}\), where \(n\) is the dimension of the "object" in question, a string in our case (could be \(n = 0\) (particle), \(n = 2\) (membrane)). This implies the ability to gauge out \(g_{ij}\), so that it’s equal to \(g^{ij} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \).
Closed String Quantization: (remarks)

• $n$-dim objects $\Rightarrow$ Action: invariant $(n+1)$-dim volume. Describing the intrinsic geometry of this $(n+1)$-dim manifold by the metric $h_{\alpha\beta}(\sigma)$ and $h = \left| \det(h_{\alpha\beta}) \right|$, 

$$S = -\frac{(2\pi \alpha')^{-1}}{2} \int h^{\alpha\beta}(\sigma) \partial_\alpha X^\mu \partial_\beta X^\nu g_{\mu\nu}(X) \sqrt{h} \, d\tau \, d\sigma,$$

and, as seen above, conformal [Weyl] invariance only appears for $n = 1$ objects, i.e., strings! 😊

• Higher dimensional objects (membranes and so on): The action above defines a $(n+1)$-dim QFT, which, by power counting, is renormalizable for $n = 1$ and unrenormalizable for $n > 1$! $\Rightarrow$ making sense of it is as difficult as making sense of GR as a quantum theory! $\Rightarrow$ membranes or higher-dim objects wouldn’t be a promising start towards quantum gravity! 😊

(“Who will save the day?!”)
Closed String Quantization: (cont’d)

• Closed Strings, Periodic Boundary Conditions: Equations of Motion ⇒ mode expansion:

\[ X^\mu = x^\mu + \frac{\alpha'}{2} p^\mu \log(|z|^2) + \]
\[ + i \sqrt{\frac{\alpha'}{2}} \sum_{n \neq 0} \left( \frac{\alpha_n^\mu}{n} z^{-n} + \frac{\tilde{\alpha}_n^\mu}{n} \bar{z}^{-n} \right); \]

where \( z \equiv e^{i(t-\sigma)} = e^{\tau - i\sigma} \). \( \alpha_n^\mu \): left-mover, \( \tilde{\alpha}_n^\mu \): right-mover.

• SuperStrings: need 2-dimensional fermions ⇒ \( \Psi^\mu = \begin{pmatrix} \psi^\mu \\ \bar{\psi}_\mu \end{pmatrix} \): Majorana (real) fermions, satisfying either periodic (Ramond) or anti-periodic (Neveu-Schwarz) boundary conditions: \( \psi^\mu(\sigma + 2\pi) = \pm \psi^\mu(\sigma) \) (chirality). (Analogously for right-movers.) Mode expansions are given by,

\[ \Psi^\mu = \begin{cases} 
\sum_{m \in \mathbb{Z}} d_m^\mu z^{-m} & : R \\
\sum_{r \in \mathbb{Z} + 1/2} b_r^\mu \bar{z}^{-r} & : NS 
\end{cases} \]
Closed String Quantization: (cont’d)

- Commutation Relations:

\[
[\alpha^\mu_m, \alpha^n_\nu] = m \eta^{\mu\nu} \delta_{m+n,0}
\]

\[
\{ b^\mu_r, b^\nu_s \} = \eta^{\mu\nu} \delta_{r+s,0} ; \quad \{ d^\mu_m, d^\nu_n \} = \eta^{\mu\nu} \delta_{m+n,0} .
\]

- Critical Dimension: 10 = 1 + 9 (validity of the no-ghost theorem — negative norm states);

- Ground states: Exist for left- and right-movers \( \Rightarrow \) towers of massive states;

- Closed strings: multiply left- and right-mover (both have SUSY \( \Rightarrow N = 2 \) in 10-dim),

- Opposite chiralities \( (\psi_+, \tilde{\psi}_-) \): vector-like theory — Type IIA superstring;

- Same chiralities \( (\psi_+, \tilde{\psi}_+) \): chiral theory — Type IIB superstring.
Closed String Quantization: (cont’d)

- **Heterotic String**: left-mover = superstring and right-mover = bosonic string (or vice-versa) (only $N = 1$ SUSY in 10-dim) ⇒ Consistency implies that,
  - $\text{SO}(32)$;
  - $E_8 \times E_8$.

Thus, this far there are 4 (consistent) superstring theories!

The fifth and final consistent superstring theory is the *open string*, discussed below.
**Open String:**

The variation of the action, when taking open strings into account, yields boundary terms, that must be vanishing,

\[ \delta X^\mu \partial_\sigma X_\mu = 0 , \]

\[ \Rightarrow \partial_\sigma X^\mu = 0 \text{ (Neumann)} \text{ or } \delta X^\mu = 0 \text{ (Dirichlet)}. \]

- **Neumann BC:** open string,
  - Same mode expansion with the constraint that right- and left-movers are the same!
  - **Chan-Paton factors:** internal degrees-of-freedom at the end points.
  - Gauge group: SO(32) (consistency) \( \Rightarrow \text{no-orientation}. \)
  - Constraint: \( N = 1 \text{ SUSY}. \)
  - SO(32) Type I string theory! 😊 (The fifth and final one!)
Open String: (cont’d)

- **Dirichlet BC**: violates translational invariance ⇒ momentum conservation!
  
  - **Boundary**: Has to have something to conserve momentum ⇒ **D-Branes**! 😊

- **D-Branes**: defined by the property that open strings can attach to it! (This breaks half of the SUSY.)
  
  - Just the BC for the bosonic part ($X^\mu$) has been discussed so far.

  - The **same** can be done for fermions, yielding consistent definitions of D-branes!

- Not only D-branes *should* be considered, they *must* be taken into account ⇔ T-Duality!
T-Duality of the Closed String:

- Closed String compactified on a circle of radius \( R \):
  \[
  X \simeq X + 2\pi R .
  \]
- Momentum quantization: \( p = n/R, \ n \in \mathbb{Z} \).
- Wrapping of the circle: \( X(\sigma+2\pi) = X(\sigma)+2\pi R w, \ w \in \mathbb{Z} \).

- Effects of \( w \neq 0 \): Separate mode expansions,
  \[
  X_L(z) = x_L - i \frac{\alpha'}{2} p_L \log(z) + i \sqrt{\frac{\alpha'}{2}} \sum_{n \neq 0} \frac{\alpha_n}{n} z^{-n} ,
  \]
  \[
  X_R(\bar{z}) = x_R - i \frac{\alpha'}{2} p_R \log(\bar{z}) + i \sqrt{\frac{\alpha'}{2}} \sum_{n \neq 0} \tilde{\alpha}_n \bar{z}^{-n} ,
  \]
T-Duality of the Closed String: (cont’d)

with,

\[ p_L = \frac{n}{R} + \frac{w R}{\alpha'} \]

\[ p_R = \frac{n}{R} - \frac{w R}{\alpha'} \]

- Hamiltonian constraint (2-d gravity): spectrum of the theory (Number operators: left-mover: \( N \equiv \alpha^\mu_{-n} \alpha_n \mu \), right-mover: \( \tilde{N} \equiv \tilde{\alpha}^\mu_{-n} \tilde{\alpha}_n \mu \)),

\[ N - \tilde{N} + nw = 0 \]

- Symmetry of the spectrum: \( R \mapsto R' = \alpha'/R \) and \( n \mapsto w \), which means \( p_L \mapsto p_L \) and \( p_R \mapsto -p_R \).

- Extending this symmetry: \( X(z, \bar{z}) = X_L(z) + X_R(\bar{z}) \mapsto X'(z, \bar{z}) = X_L(z) - X_R(\bar{z}) \mapsto T\text{-Duality!} 😊 \)
T-Duality of the Closed String: (cont’d)

- World-sheet SUSY under T-duality:
  \[ \tilde{\psi}'(\bar{z}) \mapsto -\tilde{\psi}(\bar{z}) . \]
  - Flipped chirality (right-mover)!

- Therefore, **T-Duality**: Type IIA \(\leftrightarrow\) Type IIB!

- Note that, the Euler-Lagrange eq derived from the string action is simply the 2-d wave equation,
  \[
  \Box X^\mu \equiv \left( \frac{\partial^2}{\partial \sigma^2} - \frac{\partial^2}{\partial \tau^2} \right) X^\mu = 0 ,
  \]
  \[
  X^\mu(\sigma) = X^\mu_R(\tau - \sigma) + X^\mu_L(\tau + \sigma) ,
  \]
  \[\Rightarrow \mathbb{T}[X^\mu(\sigma)] \equiv X^\mu_R(\tau - \sigma) - X^\mu_L(\tau + \sigma) .\]
**T-Duality of the Open String:**

- Natural to consider the same transformation for open strings;

- "What is T-duality?!" It is the exchange of $\tau$ and $\sigma$! Or a $\pi/2$ rotation on the worldsheet,

![T-duality Diagram]

- Thus, $\mathbb{T}[\partial_\sigma X^\mu = 0] = [\partial_\tau X^\mu = 0]$: Exchange of the Neumann and Dirichlet BC!!!
T-Duality of the Open String: (cont’d)

- No way to escape both boundary conditions!
- ∃ something that fixes the end-points of open strings: **D-branes**! 😊 (When they have $p$ spatial dimensions, they’re called **D$p$-branes**.)

- Preserve (half of) SUSY [BPS objects];

- No force between parallel D$p$-branes [remaining SUSY and cancellation of (NS,NS) and (R,R) bosons];

- Carry RR charge [couple to RR forms in type II theories — RR forms of odd (even) rank exist in type IIA (IIB) ⇒ type IIA (IIB) contains D$p$-branes with even (odd) $p$].

- Open strings attach to D-branes ⇒ effective theory on the D-brane is a gauge theory!
**T-Duality of the Unoriented String:**

- **Closed String:** Worldsheets parity transformation,

\[ \Omega : \sigma \mapsto 2\pi - \sigma , \]

and require that **all** the states in the theory obey \( \Omega = +1 \).

- Physically \( \Rightarrow \) **all** the states are invariant under **orientation reversal:** Theory of Unoriented Strings!

- Under T-duality \((m: \text{ space index})\):

\[ \mathbb{T}[X^m(z, \bar{z})] = X^m_L(z) - X^m_R(\bar{z}) , \]

\[ \Omega \left\{ \mathbb{T}[X^m(z, \bar{z})] \right\} = -\mathbb{T}[X^m(z, \bar{z})] \]

- This **defines** the unoriented string.

- **Target Space:** not a circle, but a **half line**, \( S^1/\mathbb{Z}_2 \), called an **orientifold**.
**T-Duality of the Unoriented String: (cont’d)**

- **Simple Example:**
  
  - Circular coord, \(-\pi R \leq x \leq \pi R\),
  
  - Above \(\mathbb{Z}_2\) identification comes from the invariance under the \(\Omega\) transformation (parity on periodic BC).
  
  - Space becomes: \(0 \leq x \leq \pi R\).

- **Orientifold fixed planes:** at \(x = 0\) and \(x = \pi R\) (rigid, fixed planes; unlike D-branes which are dynamical).

- Close to the orientifold (fixed planes): unoriented open strings and closed strings \(\Rightarrow\) Type I may be regarded as Type IIB on the orientifold!
Branes: An Introduction

Part II

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Outline:

- Strong Coupling Limit of Type II and S-duality;

- D-branes & SUGRA;

- AdS/CFT correspondence;

- Dual Gravity description of NC SYM;

- Cardy-Verlinde formula;

- Discussions.
**Strong Coupling Limit of Type II and S-Duality:**

- **Type IIA:** Dp-branes, even $p$. Consider the D0-brane.

  1. Tension/Energy: $E_0 = 1/g \sqrt{\alpha'}$, where $g$ is the string coupling constant.

  2. BPS (SUSY representation): Energy of $n$ bound states is $E_n = n E_0$. (No binding energy.)

  3. BPS: $E_n$ not changed by quantum corrections.

- **Perturbation Theory:** D0-branes can be neglected (mass inversely proportional to $g$: heavy),

- **Strong Coupling:** light objects! With a [huge] tower of massive states given by $E_n$!
Strong Coupling Limit of Type II and S-Duality: (cont’d)

- Tower of light massive states: KK-modes (Kaluza-Klein) from 11-dim! ⇐ First signal of M-Theory! 😊

- Comparison of the low-energy, effective actions (massless) — (R, R) and (NS, NS) ⇒ 11-dim metric in terms of type IIA fields.

  - 11-dim radius: \( R_{11} = e^{2 \phi} = g^{\frac{2}{3}} \). This solves a couple of questions,

    1. Perturbation in \( g \equiv \) expansion around \( R_{11} = 0! \) ⇐ Reason why string perturbation cannot see the 11th dimension!

    2. Masses of KK-modes: \( \sim \frac{1}{g} \) ⇒ Matches those of D0-branes! They are also BPS states with RR charge, consistent with D0-branes.
**Strong Coupling Limit of Type II and S-Duality:** (cont’d)

- **Strong coupling limit:** \( g \to \infty \Rightarrow R_{11} \to \infty \), is an 11-dim SUSY theory with gravity! The low-energy, effective, theory (containing only massless degrees) must be the 11-dim SUGRA, which is the **only** theory with this property! 😊

- **Type IIB:** Strong coupling limit yields **itself**!

- **S-Duality:** Strong/Weak coupling limits!

- **Summary:**
Summary & Overall Picture:

- Thus, the overall picture of this zoo of theories and dualities is as follows:

![Diagram showing M Theory, Type I, Type IIA, Type IIB, and their dualities.]

- Hořava-Witten conjecture:

  $$\text{M-Theory}/(S^1/\mathbb{Z}_2) \simeq \text{Heterotic } E_8 \times E_8$$

  That is, M-Theory compactified on $S^1 \times S^1/\mathbb{Z}_2$ is the Heterotic $E_8 \times E_8$ superstring theory! [Note that $S^1/\mathbb{Z}_2$ is an orientifold.]

- Thus, all superstrings are related to each other either by S- or T-Duality!
D-branes & SUGRA:

- M-theory $\xrightarrow{S}$ 11-dim SUGRA: D-branes are classical soliton solutions!

- D-branes:
  - Previously: Described in terms of the perturbative string picture,
  - Now: SUGRA classical solitonic solutions.

- "Why?!" (classical solutions are D-branes)
  - They [the solutions] are spatially $p$-dim extended BPS objects,
  - Carry RR charge.
D-branes & SUGRA: (cont’d)

- D-brane charges are quantized.

- Constructing a 5-dim black hole SUGRA solution ⇒ Bekenstein-Hawking entropy is quantized! ☺ (The entropy is proportional to the D-brane charge.)

- Statistical description of the black hole entropy: the information is stored on the brane (horizon) ⇐ Type of Holographic principle! (Information stored on the boundary!) ☺
AdS\(_{p+2}/CFT\}_{p+1} \ \text{Correspondence/Duality:}

- **Known results:** (AdS-type brane SUGRA solution: CFT behaviour!)

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<td>AdS(_3/CFT_2)</td>
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- **Agreement of**
  1. Symmetries,
  2. Spectrum,

- **Properties of gravity:** evolution of Hawking radiation & information loss can be studied by “well behaved”, unitary CFT ⇒ No Information Loss paradox!

- **Formulate string theory in terms of Field Theory:** Matrix Theory!
Dual gravity and NC SYM:

- **Constant Background B field** on the D-brane:
  *Non-Commutative Super-Yang-Mills theory* on the D-brane!

- Generalization of AdS/CFT is possible! 😊

- Theory can be described by *Non-Commutative Field Theory* or *Dual Gravity*.

- *Dual Gravity*: asymptotically AdS but significantly different in the short distance.

- Non-commutative space: Discretized, \([x^\mu, x^\nu] = i e^{\mu\nu}\).
Cardy-Verlinde formula:

- **Holographic Principle:** given a volume $V$, the state of maximal entropy is given by the largest black hole that fits inside $V!$

- Bekenstein-Hawking entropy: $S \leq \frac{A}{4G}$.

- Verlinde:

  - Einstein Universe: $ds^2 = -dt^2 + R^2 d\Omega^2_n \Rightarrow S = \frac{2\pi R}{n} \sqrt{E_c (2E - E_c)}$, where $E$ is the total energy and $E_c$ is the Casimir energy.

  - $(n + 1)$-dim closed Universe: $H = \dot{R}/R$ is the Hubble parameter, $E$ is the total energy of matter filling the Universe, $V$ the Universe volume and $G_n$ is the $(n + 1)$-dim gravitational constant $\Rightarrow$
Cardy-Verlinde formula: \( \text{(cont’d)} \)

1. Bekenstein-Verlinde bound:

\[
S_{BV} = \frac{2\pi}{n} E R ,
\]

system with limited self-energy \( \Rightarrow \) the total \( S \) is less than the energy times the linear size of the system.

2. Bekenstein-Hawking bound:

\[
S_{BH} = (n - 1) \frac{V}{4 G_n R} ,
\]

the black hole entropy is bounded by the area.

3. Hubble bound:

\[
S_H = (n - 1) \frac{H V}{4 G_n} ,
\]

maximal entropy is given by a black hole the size of the Hubble horizon.
Cardy-Verlinde formula: (cont’d)

- **Critical Point:** \( HR = 1 \Rightarrow 3 \) entropy bounds coincide!

- Define \( E_{BH} \) such that,

\[
S_{BH} \equiv \frac{2\pi E_{BH} R}{n} = \frac{(n - 1)V}{4G_n R},
\]

\[
\Rightarrow S_H = \frac{2\pi R}{n} \sqrt{E_{BH} (2E - E_{BH})}!
\]

which is of the same form as the Cardy-Verlinde formula! 😊

- Consequence of the **Holographic Principle**!
Discussions:

- **String theory:**
  - Not a theory of strings only! But a theory of many *extended* objects!
  - Intricately combined to exhibit its appearance as various string theories on perturbative vacua!

- **Under this view: D-branes play a major role!**
  - Shed further light on M-Theory. 😊
  - Non-perturbative study of,
    * Gauge theories,
    * Non-commutative field theory!

- **M-Theory: unifies all string theories (string vacua)! 😊**