Branes: An Introduction
Part I

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Outline:

- Overview;
- Review of Basic Results:
  - Closed String Quantization;
  - Open String;
- T-duality:
  - Closed String;
  - Open String;
  - Unoriented String.

Overview:

"Once upon a time. . . " the world of physics was divided into two categories:

- Small: Quantum Mechanics, Quantum Field Theory;

- Large: General Relativity (Einstein's theory of gravity).

Nowadays, those two pillars of modern physics are in conflict. . . One that is the modern version of earlier contradictions that lead to radical changes in the 20th century physics!

Overview: (cont'd)

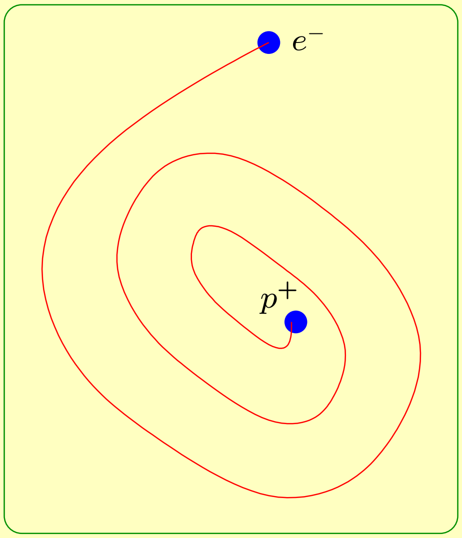
Back in the *"dark days"*, the inverse square law of gravity and electromagnetism where very much alike:

$$F_{\mathsf{g}} = G\,\frac{M\,m}{r^2}\,, \qquad F_{\mathsf{e}} = K\,\frac{Q\,q}{r^2}\,.$$

This worked just fine for about 200 years... ☺ [It's worth noting at this point that both of those "laws" follow from Gauss' Law and spherical symmetry! Only the charges and the proportionality constants are different.]

However, when the electron was discovered... hell broke loose! ☹ And, ever since, physicists have had to deal with the singularity at $r = 0$!

Overview: (cont'd)



This seemed to imply that, e.g., an atom would survive for about 10^{-9} seconds!

However, this problem was solved with the aid of the **uncertainty principle!**

What it does is that it "smears out" the singularity at the origin (everything gets a bit fuzzy).

It turns out that this just doesn't do it for gravity! ☹ The analogy between gravity and eletromagnetism isn't as close as the inverse square law suggests... And, in actuallity, since 1916, gravity is described by Einstein's **General Relativity**. (Thus, there is **no** inverse square law!) And, basically, the non-linearity of GR is what prevents the uncertainty principle from working.

Overview: (cont'd)

This leads us to **string theory**: During the 70's, physicists discovered that this problem can be overcome in the stringy framework.

Roughly speaking, one reinterprets an elementary particle as a vibrating string/loop. The "roughly" is because (among other things) one should include **quantum uncertainty** on both of them: the particle and the string side!

Strings: many modes of oscillation/harmonics (representing different particles) $\Rightarrow$ single string describes all (a huge assortment of particles) at once!

$\therefore$ upon quantization of the string and computation of the "spectrum" $\Rightarrow$ find that one of these particles has the properties to be a **"graviton"**, the quantum of gravity!

Overview: (cont'd)

- Conventional QFT: Gravity is **impossible**;
- String Theory:
  - Gravity is **required**!
  - Other modes: photon, electron, muon, quarks, neutrinos, etc. . .

That is: Very simple assumptions $\Rightarrow$ "everything" neatly fits into a string! ☺

Overview: (cont'd)

Among the fundamental differences between [ordinary] quantum theory and string theory, these are eye-popping ones:

- Vertices:
  - QFT: the interactions have to be described,
  - String: once the motion of a free string is understood, understanding the interactions is automatic, since the picture looks everywhere locally the same.

- Infinities:
  - QFT: Coalescing of the vertices $\Rightarrow$ product of δ-functions (iff inverse square law — point-like interactions).
  - String: **no vertices** $\Rightarrow$ finite!

Overview: (cont'd)

- QFT:
  - Free motion: doesn't really contain much information about spacetime;
  - Interaction (vertex): depends on knowing exactly where (product of δ-functions) they happen!
- String: this last piece is missing $\Rightarrow$ **never** learns precisely what the spacetime is: **New** source of fuzziness that goes beyond quantum mechanical uncertainty,
  - $\hbar$: quantum mechanical $\longrightarrow$ bound on the precision of "trajectory of a particle",
  - $\boldsymbol{\alpha}$: size of string $\longrightarrow$ bound on the description of spacetime!

Overview: (cont'd)

String theory: 3 general predictions:

1. Gravity (Einstein's GR);
2. Gauge Symmetry (bread 'n butter of QFT); &
3. SuperSymmetry:
   - alongside the usual ("bose") dimensions of spacetime (t, x, y, z), there would be "fermi" dimensions, θ^α. It predicts new particles $\Longleftrightarrow$ oscillations in the dimension.

One does need some knowledge about the vacuum, though.

Overview: (cont'd)

	Particles	Waves
Old	Quantum Mechanics	GR and QFT
New	String Theory	(???)

This problem has been around for quite a while. . . but its outlook has changed drastically in the last couple of years: **Duality!!!**

Duality:

- Old version: Symmetry between $\vec{E}$ and $\vec{B}$ which holds in vacuum ($c = 1$),

$$\nabla \cdot \vec{E} = 0 \, , \quad \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \, ,$$

$$\nabla \cdot \vec{B} = 0 \, , \quad \nabla \times \vec{B} = \frac{\partial \vec{E}}{\partial t} \, ,$$

$$\vec{E} \mapsto \vec{B} \, , \quad \vec{B} \mapsto -\vec{E} \, .$$

Overview: (cont'd)

It is spoiled in nature due to the absence of a **magnetic monopole**.

Quantum Mechanics seems to make duality impossible: Role of the vector potential, $\vec{A}$.

Olive & Montonen: strong/weak coupling duality! (Swaping the electric charge with a magnetic monopole — supposed to exist.)

Limitations in our abilities in physics: *"We are able to compute what happens only when the charge, e, is much less than one!"* — Perturbation Theory!

Duality: exchange e and $1/e$ — both of them cannot be much less than one!

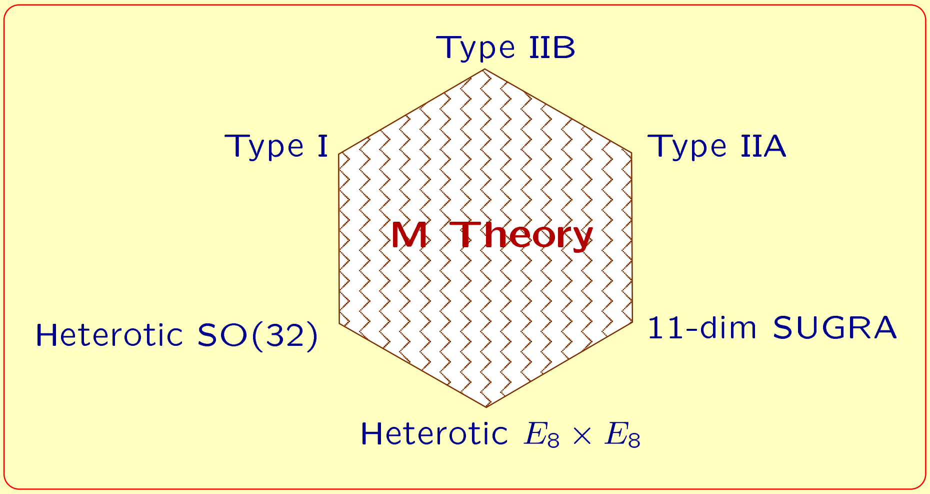
New understanding of duality (non-perturbative): Severe implications for QFT and String Theory.

Overview: (cont'd)

E.g.:

- QFT: Quark confinement (related to more standard physics via duality — strong vs weak coupling),

- ST: There is only **one** ST!!! ☺ ⇒ Type I, Type IIA, Type IIB, Heterotic SO(32), $E_8 \times E_8$ and 11-dim SUGRA are different limiting cases of one theory.



Overview: (cont'd)

Better understanding of String Theory: *Strings are the "first among equals" and share the stage with "D-branes" — they are related by duality!*

D-branes successes:

1. Black Hole entropy;
2. SuperSymmetric field theories;
3. Non-supersymmetric field theories;
4. Cosmological problems: the Hierarchy problem;
5. Non-commutative spacetime...

Closed String Quantization: (quick review)

- Worldsheet action:

$$S = -\frac{1}{2}\frac{1}{2\pi\,\alpha'}\int_{\mathcal{W}} \sqrt{-g}\, g^{ij}\,\partial_i X^\mu\,\partial_j X^\nu\,\eta_{\mu\nu}\,dt\,d\sigma\ ;$$

  where $(1/2\pi\,\alpha')$ is the string tension; $i,j = 0,1$ are the worldsheet coordinates (τ,σ); and X^μ are the spacetime coordinates, with $\mu,\nu = 0,\ldots,D-1$. (This can also be understood as 2-dimentional gravity coupled to "matter" X^μ.)

  – 2-dim reparametrization invariance: Just like in GR, $\sqrt{-g}\,d\tau\,d\sigma$ is the invariant volume and $g^{ij}\,\partial_i X^\mu\,\partial_j X_\mu$ is properly contracted.

  – Conformal [Weyl] invariance: $g_{ij} \mapsto f(\sigma)\,g_{ij} \Rightarrow \sqrt{-g}\,g^{ij} \mapsto f^{\frac{1}{2}(n+1)-1}\sqrt{-g}\,g^{ij}$, where n is the dimension of the "object" in question, a string in our case (could be $n=0$ (particle), $n=2$ (membrane)). This implies the ability to gauge out g_{ij}, so that it's equal to $g^{ij} = \left(\begin{smallmatrix}-1 & 0\\ 0 & 1\end{smallmatrix}\right)$.

Closed String Quantization: (remarks)

- n-dim objects $\Rightarrow$ Action: invariant $(n+1)$-dim volume. Describing the intrinsic geometry of this $(n+1)$-dim manifold by the metric $h_{\alpha\beta}(\sigma)$ and $h = \Big|\text{det}(h_{\alpha\beta})\Big|$,

$$S = -\frac{(2\pi\,\alpha')^{-1}}{2} \int h^{\alpha\beta}(\sigma)\,\partial_\alpha X^\mu\,\partial_\beta X^\nu\, g_{\mu\nu}(X)\,\sqrt{h}\,d\tau\,d\sigma\;,$$

  and, as seen above, conformal [Weyl] invariance only appears for $n = 1$ objects, i.e., strings! ☺

- Higher dimensional objects (membranes and so on): The action above defines a $(n+1)$-dim QFT, which, by power counting, is renormalizable for $n = 1$ and unrenormalizable for $n > 1$! $\Rightarrow$ making sense of it is as difficult as making sense of GR as a quantum theory! $\Rightarrow$ membranes or higher-dim objects wouldn't be a promising start towards quantum gravity! ☹ *("Who will save the day?!")*

Closed String Quantization: (cont'd)

- Closed Strings, Periodic Boundary Conditions: Equations of Motion ⇒ *mode expansion*:

$$X^\mu = x^\mu + \frac{\alpha'}{2} p^\mu \log(|z|^2) + \\ + i\sqrt{\frac{\alpha'}{2}} \sum_{n\neq 0} \left(\frac{\alpha_n^\mu}{n} z^{-n} + \frac{\tilde{\alpha}_n^\mu}{n} \bar{z}^{-n} \right) ;$$

  where $z \equiv e^{i(t-\sigma)} = e^{\tau - i\sigma}$. α_n^μ: *left-mover*, $\tilde{\alpha}_n^\mu$: *right-mover*.

- SuperStrings: need *2-dimensional fermions* ⇒ $\Psi^\mu = \binom{\psi^\mu}{\tilde{\psi}^\mu}$: Majorana (real) fermions, satisfying either *periodic* (Ramond) or *anti-periodic* (Neveu-Schwarz) boundary conditions: $\psi^\mu(\sigma + 2\pi) = \pm\psi^\mu(\sigma)$ (chirality). (Analogously for right-movers.) *Mode expansions* are given by,

$$\Psi^\mu = \begin{cases} \sum_{m\in\mathbb{Z}} d_m^\mu \, z^{-m} & : \text{R} \\ \\ \sum_{r\in\mathbb{Z}+1/2} b_r^\mu \, z^{-r} & : \text{NS} \end{cases}$$

Closed String Quantization: (cont'd)

- Commutation Relations:

$$[\,\alpha_m^\mu, \alpha_n^\nu\,] = m\,\eta^{\mu\nu}\,\delta_{m+n,0}$$

$$\{\,b_r^\mu, b_s^\nu\,\} = \eta^{\mu\nu}\,\delta_{r+s,0}\;; \quad \{\,d_m^\mu, d_n^\nu\,\} = \eta^{\mu\nu}\,\delta_{m+n,0}\,.$$

- Critical Dimension: $10 = 1 + 9$ (validity of the no-ghost theorem — negative norm states);

- Ground states: Exist for left- and right-movers $\Rightarrow$ towers of massive states;

- Closed strings: multiply left- and right-mover (both have SUSY $\Rightarrow$ $N = 2$ in 10-dim),
  - Opposite chiralities $(\psi_+, \tilde{\psi}_-)$: vector-like theory — *Type IIA* superstring;
  - Same chiralities $(\psi_+, \tilde{\psi}_+)$: chiral theory — *Type IIB* superstring.

Closed String Quantization: (cont'd)

- *Heterotic String*: left-mover = superstring and right-mover = bosonic string (or vice-versa) (only $N = 1$ SUSY in 10-dim) ⇒ Consistency implies that,
  - SO(32);
  - $E_8 \times E_8$.

Thus, this far there are 4 (consistent) superstring theories!

The fifth and final consistent superstring theory is the *open string*, discussed below.

Open String:

The variation of the action, when taking open strings into account, yields boundary terms, that must be vanishing,

$$\delta X^\mu \partial_\sigma X_\mu = 0 \ ,$$

$\Rightarrow \partial_\sigma X^\mu = 0$ (Neumann) or $\delta X^\mu = 0$ (Dirichlet).

- Neumann BC: open string,
  - Same mode expansion with the constraint that right- and left-movers are the **same**!
  - *Chan-Paton factors:* **internal** degrees-of-freedom at the **end** points.
  - Gauge group: SO(32) (consistency) ⇒ **no-orientation**.
  - Constraint: $N = 1$ SUSY.
  - SO(32) Type I string theory! ☺ (The fifth and final one!)

Open String: (cont'd)

- Dirichlet BC: violates translational invariance $\Rightarrow$ momentum conservation!
  - Boundary: Has to have something to conserve momentum $\Rightarrow$ **D-Branes**! ☺
- **D-Branes**: defined by the property that open strings can attach to it! (This breaks half of the SUSY.)
  - Just the BC for the bosonic part (X^μ) has been discussed so far.
  - The **same** can be done for fermions, yielding consistent definitions of D-branes!
- Not only D-branes *should* be considered, they *must* be taken into account $\Leftarrow$ T-Duality!

T-Duality of the Closed String:

- Closed String compactified on a circle of radius R:

$$X \simeq X + 2\pi\, R\,.$$

  – Momentum quantization: $p = n/R,\ n \in \mathbb{Z}$.

  – Wrapping of the circle: $X(\sigma{+}2\pi) = X(\sigma) + 2\pi\, R\, w,\ w \in \mathbb{Z}$.

- Effects of $w \neq 0$: Separate mode expansions,

$$X_L(z) = x_L - i\,\frac{\alpha'}{2}\,p_L\,\log(z) + i\sqrt{\frac{\alpha'}{2}}\sum_{n\neq 0}\frac{\alpha_n}{n}\,z^{-n}\,,$$

$$X_R(\bar{z}) = x_R - i\,\frac{\alpha'}{2}\,p_R\,\log(\bar{z}) + i\sqrt{\frac{\alpha'}{2}}\sum_{n\neq 0}\frac{\tilde{\alpha}_n}{n}\,\bar{z}^{-n}\,,$$

T-Duality of the Closed String: (cont'd)

with,

$$p_L = \frac{n}{R} + \frac{w\,R}{\alpha'} \;,$$

$$p_R = \frac{n}{R} - \frac{w\,R}{\alpha'} \;.$$

- Hamiltonian constraint (2-d gravity): spectrum of the theory (Number operators: left-mover: $N \equiv \alpha^{\mu}_{-n}\,\alpha_{n\mu}$, right-mover: $\tilde{N} \equiv \tilde{\alpha}^{\mu}_{-n}\,\tilde{\alpha}_{n\mu}$),

$$N - \tilde{N} + n\,w = 0 \;.$$

- Symmetry of the spectrum: $R \mapsto R' = \alpha'/R$ and $n \mapsto w$, which means $p_L \mapsto p_L$ and $p_R \mapsto -p_R$.

- Extending this symmetry: $X(z,\bar{z}) = X_L(z) + X_R(\bar{z}) \mapsto X'(z,\bar{z}) = X_L(z) - X_R(\bar{z}) \Rightarrow$

T-Duality! ☺

T-Duality of the Closed String: (cont'd)

- World-sheet SUSY under T-duality:

$$\tilde{\psi}'(\bar{z}) \mapsto -\tilde{\psi}(\bar{z}) \; .$$

  - Flipped chirality (right-mover)!

- Therefore, **T-Duality:** Type IIA ↔ Type IIB! ☺

- Note that, the Euler-Lagrange eq derived from the string action is simply the 2-d wave equation,

$$\Box\, X^\mu \equiv \left(\frac{\partial^2}{\partial\sigma^2} - \frac{\partial^2}{\partial\tau^2}\right) X^\mu = 0 \; ,$$

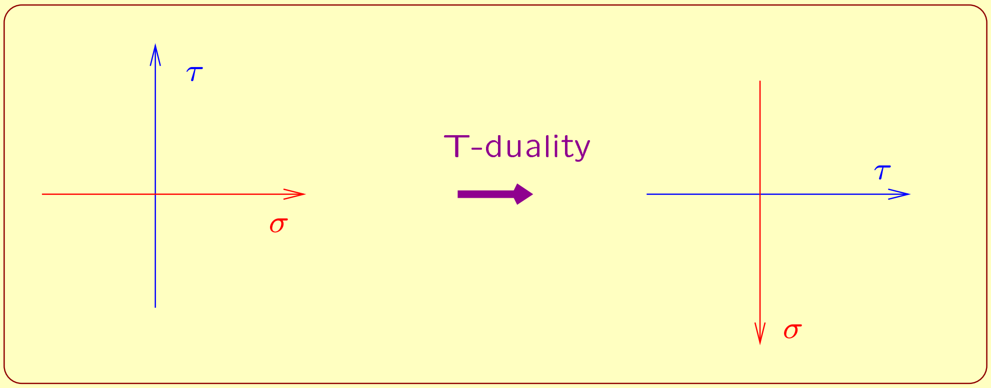
$$X^\mu(\sigma) = X_R^\mu(\tau-\sigma) + X_L^\mu(\tau+\sigma) \; ,$$

$$\Rightarrow \mathbb{T}[X^\mu(\sigma)] \equiv X_R^\mu(\tau-\sigma) - X_L^\mu(\tau+\sigma) \; .$$

T-Duality of the Open String:

- Natural to consider the same transformation for open strings;

- *"What is T-duality?!"* It is the exchange of τ and σ! Or a $\pi/2$ rotation on the worldsheet,

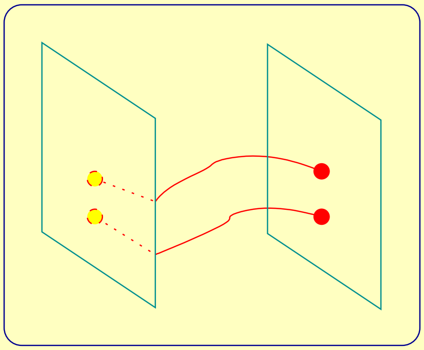


- Thus, $\mathbb{T}[\partial_\sigma X^\mu = 0] = [\partial_\tau X^\mu = 0]$: **Exchange** of the Neumann and Dirichlet BC!!!

T-Duality of the Open String: (cont'd)

- No way to escape **both** boundary conditions!

- ∃ something that fixes the end-points of open strings: **D-branes**! ☺ (When they have p spatial dimensions, they're called **Dp-branes**.)



  - Preserve (half of) SUSY [BPS objects];

  - No force between parallel Dp-branes [remaining SUSY and cancellation of (NS,NS) and (R,R) bosons];

  - Carry RR charge [couple to RR forms in type II theories — RR forms of odd (even) rank exist in type IIA (IIB) ⇒ type IIA (IIB) contains Dp-branes with even (odd) p].

  - Open strings attach to D-branes ⇒ effective theory on the D-brane is a gauge theory!

T-Duality of the Unoriented String:

- Closed String: Worldsheet parity transformation,

$$\Omega : \ \sigma \mapsto 2\pi - \sigma \ ,$$

  and require that all the states in the theory obey $\Omega = +1$.

  - Physically $\Rightarrow$ all the states are invariant under orientation reversal: Theory of Unoriented Strings!

- Under T-duality (m: space index):

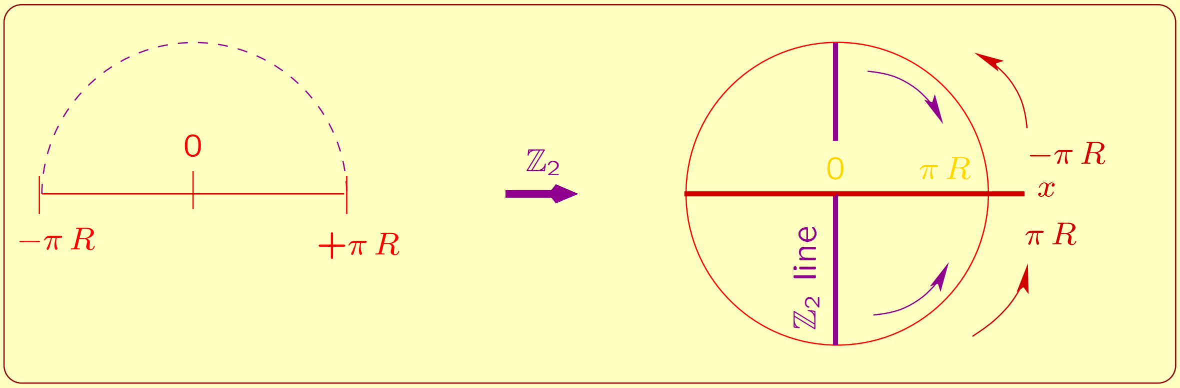
$$\mathbb{T}[X^m(z,\bar{z})] = X_L^m(z) - X_R^m(\bar{z}) \ ,$$
$$\Omega\Big\{\mathbb{T}[X^m(z,\bar{z})]\Big\} = -\mathbb{T}[X^m(z,\bar{z})]$$

  - This **defines** the unoriented string.

  - Target Space: **not** a circle, but a **half line**, $S^1/\mathbb{Z}_2$, called an **orientfold**.

T-Duality of the Unoriented String: (cont'd)

- Simple Example:
  - Circular coord, $-\pi R \leqslant x \leqslant \pi R$,
  - Above $\mathbb{Z}_2$ identification comes from the invariance under the Ω transformation (parity on periodic BC).
  - Space becomes: $0 \leqslant x \leqslant \pi R$.



  - *Orientfold fixed planes:* at $x = 0$ and $x = \pi R$ (rigid, fixed planes; unlike D-branes which are dynamical).
  - Close to the orientfold (fixed planes): unoriented open strings and closed strings $\Rightarrow$ Type I may be regarded as Type IIB on the orientfold!

Branes: An Introduction
Part II

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Outline:

- Strong Coupling Limit of Type II and S-duality;
- D-branes & SUGRA;
- AdS/CFT correspondence;
- Dual Gravity description of NC SYM;
- Cardy-Verlinde formula;
- Discussions.

Strong Coupling Limit of Type II and S-Duality:

- Type IIA: Dp-branes, even p. Consider the D0-brane.

  1. Tension/Energy: $E_0 = 1/g\sqrt{\alpha'}$, where g is the string coupling constant.

  2. BPS (SUSY representation): Energy of n bound states is $E_n = n\,E_0$. (No binding energy.)

  3. BPS: E_n not changed by quantum corrections.

- Perturbation Theory: D0-branes can be neglected (mass inversely proportional to g: heavy),

- Strong Coupling: light objects! With a [huge] tower of massive states given by E_n!

Strong Coupling Limit of Type II and S-Duality: (cont'd)

- Tower of light massive states: KK-modes (Kaluza-Klein) from 11-dim! $\Leftarrow$ First signal of **M-Theory!** ☺

- Comparison of the low-energy, effective actions (massless) — (R, R) and (NS, NS) $\Rightarrow$ 11-dim metric in terms of type IIA fields.

  – 11-dim radius: $R_{11} = e^{\frac{2}{3}\phi} = g^{\frac{2}{3}}$. This solves a couple of questions,

    1. Perturbation in $g \equiv$ expansion around $R_{11} = 0$! $\Leftarrow$ Reason why string perturbation cannot see the 11th dimension!

    2. Masses of KK-modes: $\sim \frac{1}{g}$ $\Rightarrow$ Matches those of D0-branes! They are also BPS states with RR charge, consistent with D0-branes.

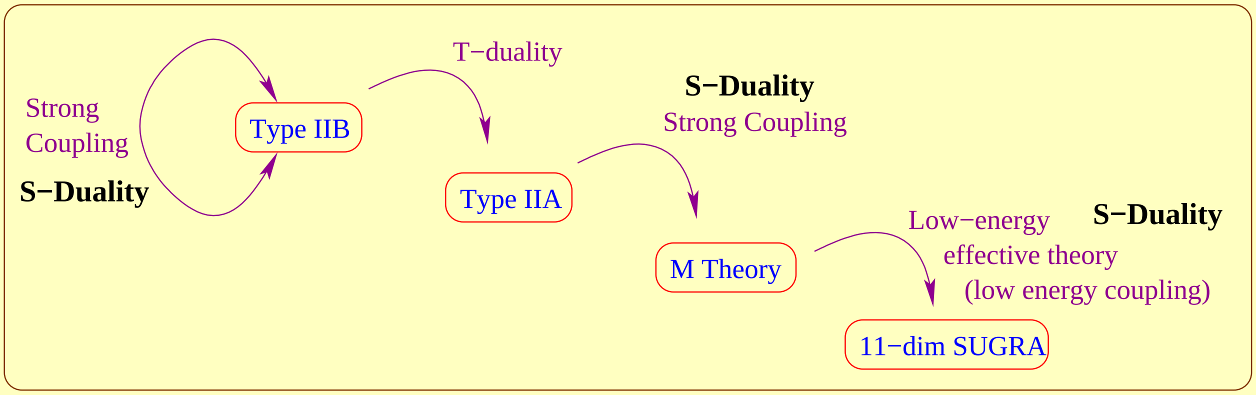
Strong Coupling Limit of Type II and S-Duality: (cont'd)

- Strong coupling limit: $g \to \infty \quad \Rightarrow R_{11} \to \infty$, is an 11-dim SUSY theory with gravity! The low-energy, effective, theory (containing only massless degrees) must be the 11-dim SUGRA, which is the only theory with this property! ☺

- Type IIB: Strong coupling limit yields **itself!**

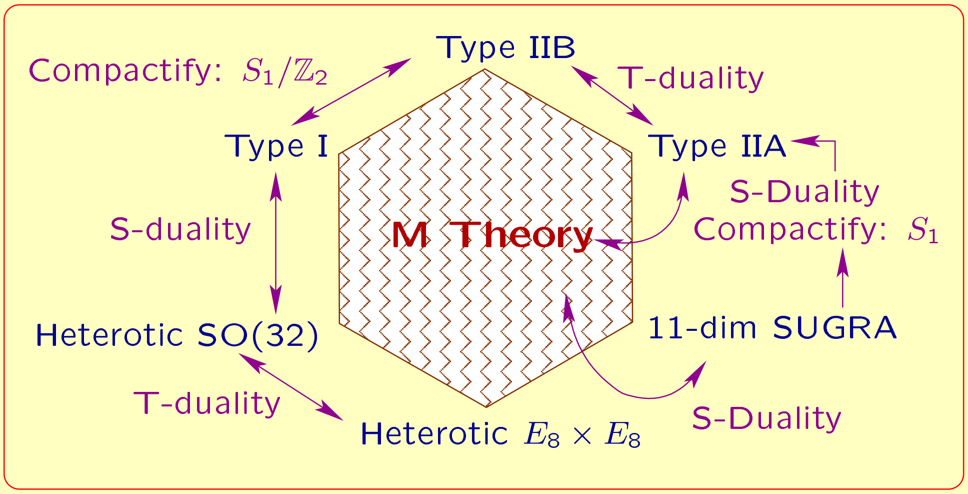
- **S-Duality: Strong/Weak coupling limits!**

- Summary:



Summary & Overall Picture:

- Thus, the overall picture of this zoo of theories and dualities is as follows:



- Hořava-Witten conjecture:

$$\text{M-Theory}/(S^1/\mathbb{Z}_2) \simeq \text{Heterotic } E_8 \times E_8 \text{ !}$$

  That is, M-Theory compactified on $S^1 \times S^1/\mathbb{Z}_2$ is the Heterotic $E_8 \times E_8$ superstring theory! [Note that $S^1/\mathbb{Z}_2$ is an orientfold.]

- Thus, all superstrings are related to each other either by S- or T-Duality!

D-branes & SUGRA:

- M-theory $\stackrel{\text{S}}{\rightarrow}$ 11-dim SUGRA: D-branes are classical soliton solutions!

- D-branes:
  - Previously: Described in terms of the perturbative string picture,
  - Now: SUGRA classical solitonic solutions.

- *"Why?!"* (classical solutions are D-branes)
  - They [the solutions] are spatially p-dim extended BPS objects,
  - Carry RR charge.

D-branes & SUGRA: (cont'd)

- D-brane charges are quantized.

- Constructing a 5-dim black hole SUGRA solution ⇒ Bekenstein-Hawking entropy is quantized! ☺ (The entropy is proportional to the D-brane charge.)

- Statistical description of the black hole entropy: the information is stored on the brane (horizon) ⇐ Type of **Holographic principle!** (Information stored on the boundary!) ☺

$\text{AdS}_{p+2}/\text{CFT}_{p+1}$ Correspondence/Duality:

- Known results: (AdS-type brane SUGRA solution: CFT behaviour!)

M-Theory	$\text{AdS}_4/\text{CFT}_3$
	$\text{AdS}_7/\text{CFT}_6$
Type IIB	$\text{AdS}_5/\text{CFT}_4$
	$\text{AdS}_3/\text{CFT}_2$

- Agreement of

  1. Symmetries,

  2. Spectrum,

  3. Operator Algebra.

- Properties of gravity: evolution of Hawking radiation & information loss can be studied by "well behaved", unitary CFT ⇒ No Information Loss paradox!

- Formulate string theory in terms of Field Theory: Matrix Theory!

Dual gravity and NC SYM:

- Constant Background B field on the D-brane: **Non-Commutative Super-Yang-Mills theory** on the D-brane!

- Generalization of AdS/CFT is possible! ☺

- Theory can be described by *Non-Commutative Field Theory* or *Dual Gravity*.

- *Dual Gravity:* asymptotically AdS but significantly different in the short distance.

- Non-commutative space: Discretized, $[\,x^\mu, x^\nu\,] = i\,c^{\mu\nu}$.

Cardy-Verlinde formula:

- **Holographic Principle:** given a volume V, the state of maximal entropy is given by the largest black hole that fits inside V!

- Bekenstein-Hawking entropy: $S \leqslant \frac{A}{4\,G}$.

- Verlinde:

  - Einstein Universe: $ds^2 = -dt^2 + R^2\, d\Omega_n^2 \Rightarrow S = \frac{2\pi\, R}{n}\sqrt{E_c\,(2\,E - E_c)}$, where E is the total energy and E_c is the Casimir energy.

  - $(n+1)$-dim closed Universe: $H = \dot{R}/R$ is the Hubble parameter, E is the total energy of matter filling the Universe, V the Universe volume and G_n is the $(n+1)$-dim gravitational constant $\Rightarrow$

Cardy-Verlinde formula: (cont'd)

1. Bekenstein-Verlinde bound:

$$S_{BV} = \frac{2\pi}{n} E\, R\,,$$

   system with limited self-energy $\Rightarrow$ the total S is less than the energy times the linear size of the system.

2. Bekenstein-Hawking bound:

$$S_{BH} = (n-1)\, \frac{V}{4\, G_n\, R}\,,$$

   the black hole entropy is bounded by the area.

3. Hubble bound:

$$S_H = (n-1)\, \frac{H\, V}{4\, G_n}\,,$$

   maximal entropy is given by a black hole the size of the Hubble horizon.

Cardy-Verlinde formula: (cont'd)

- **Critical Point:** $H\,R = 1 \Rightarrow$ 3 entropy bounds coincide!

- Define E_{BH} such that,

$$S_{BH} \equiv \frac{2\pi\, E_{BH}\, R}{n} = \frac{(n-1)\,V}{4\,G_n\,R}\,,$$
$$\Rightarrow\; S_H = \frac{2\pi\, R}{n}\sqrt{E_{BH}\,(2E - E_{BH})}\;\;!$$

  which is of the same form as the Cardy-Verlinde formula! ☺

- Consequence of the **Holographic Principle!**

Discussions:

- String theory:
  - Not a theory of strings only! But a theory of many **extended** objects!
  - Intricately combined to exhibit its appearance as various string theories on perturbative vacua!
- Under this view: D-branes play a major role!
  - Shed further light on M-Theory. ☺
  - Non-perturbative study of,
    - Gauge theories,
    - Non-commutative field theory!
- M-Theory: **unifies** all string theories (string vacua)! ☺